Spectrum-Efficiency Parametric Channel Estimation Scheme for Massive MIMO Systems

Zhen Gao, Chao Zhang, Chengran Dai, and Qian Han

***Abstract*—This paper proposes a parametric channel estimation method for massive multiple input multiple output (MIMO) systems, whereby the spatial correlation of wireless channels is exploited. For outdoor communication scenarios, most wireless channels are sparse. Meanwhile, compared with the long signal transmission distance, scale of the transmit antenna array can be negligible. Therefore, channel impulse responses (CIRs) associated with different transmit antennas usually share the very similar path delays, since channels of different transmit-receive pairs share the very similar scatterers. By exploiting the spatial common sparsity of wireless MIMO channels, we propose a parametric channel estimation method, whereby the frequency-domain pilots can be reduced significantly. The proposed method can achieve super-resolution path delays, and improve the accuracy of the channel estimation considerably. More interestingly, simulation results indicate that the required average pilot number per transmit antenna even decreases when the number of transmit antennas increases in practice.**

***Index Terms*—Channel modelling and simulation, signal processing for transmission, sparse channel estimation, massive MIMO system, multiple users (MU), OFDM**

I. Introduction

Massive multiple input multiple output (MIMO) systems employing a large number of transmit antennas at the base station to serve multiple users (MU) simultaneously can increase the spectral efficiency significantly [1]. Therefore, massive MIMO is considered to be a promising physical layer transmission technology for future communications [2].

In massive MIMO systems, accurate acquisition of the channel state information (CSI) is very essential for the sequent signal processing [4]. For massive MIMO with time division duplexing (TDD) protocol, the problem of the CSI acquisition in downlink can be relieved since the base station can obtain the CSI from uplink due to the channel reciprocity property. However, in the widely adopted communication systems with frequency division duplexing (FDD) protocol, the CSI acquisition in downlink massive MIMO is a necessary and challenging problem, since a certain user has to estimate channels associated with a large number of transmit antennas at the base station. For the conventional non-parametric MIMO channel estimation schemes, the pilot overhead for channel estimation heavily depends on the maximum delay spread of channel impulse response (CIR) and the number of transmit antennas [2]. Therefore, massive MIMO systems using the conventional non-parametric MIMO channel estimation schemes will suffer from prohibitively high pilot overhead due to numerous transmit antennas at base station.

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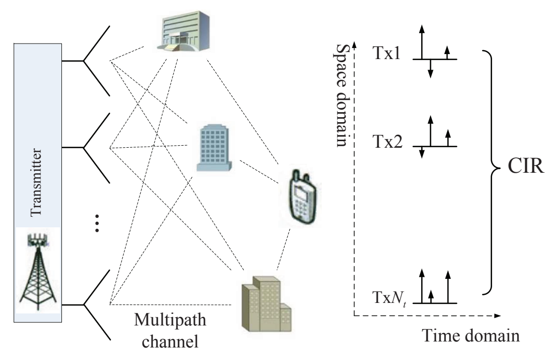

Fig. 1. Massive MIMO systems for future wireless communications.

In this paper, we propose a parametric channel estimation scheme for massive MIMO systems, whereby the spatial correlation of wireless channels are exploited to improve the accuracy of the channel estimate as well as reduce the pilot overhead. The proposed scheme can achieve the super-resolution estimation of the path delay. Moreover, with the increase of transmit antennas, to obtain the same accuracy of the channel estimate, the required number of pilots can be reduced. The main contributions of this paper are summarized as follows:

1) Due to the sparsity and the spatial correlation of wireless MIMO channels [3], [5]–[7], [9], the channel impulse responses (CIRs) of different transmit antennas share the common sparse pattern. By exploiting those two characteristics, the proposed scheme can achieve the super-resolution estimation of path delays with arbitrary values.

2) Since the sparsity of wireless channels is exploited, the proposed parametric channel estimation scheme can use less pilot overhead or enjoy higher spectral efficiency than the conventional non-parametric channel estimation schemes.

3) With the increased number of transmit antennas, to achieve the same channel estimation performance, the required number of pilots can be reduced further, or more accurate estimate of the channel can be achieved with the same number of pilots. Therefore, the proposed sparse channel estimation scheme is suited for massive MIMO systems.

II. Spatial Sparsity of Wireless MIMO Channels

The massive multiple user (MU) MIMO systems are illustrated in Fig. 1, where the base station employing N_t transmit antennas simultaneously serve multiple user equipments (UEs) with single receive antenna.

In typical outdoor communication scenarios, the CIRs are intrinsically sparse due to several scatterers. For a ceratin UE, the CIR of a certain transmit-receive antenna pair can be expressed as the tap model [10], [11], i.e.,

$$h^i(\tau) = \sum_{p=1}^{P} \alpha_p^i \delta(\tau - \tau_p^i), 1 \le i \le N_t, \tag{1}$$

where $\delta(\cdot)$ is the Dirac function, $h^i(\tau)$ is the CIR of the receive antenna associated with the ith transmit antenna, N_t is the number of transmit antennas, P is the number of resolvable propagation paths, τ_p^i is the pth path delay, α_p^i is the pth path gain.

Since the scale of transmit antenna array can be ignored in contrast with the scale of signal transmission distance, signal propagations from different transmit antennas to the same receive antenna share the very similar scatterers [9]. Therefore, as shown in Fig. 1, CIRs of different transmit antennas associated with the same receive antenna share the approximately common path delay, though the corresponding path gains are distinct owing to the small-scale fading [9]. For most communication systems, the path delay difference of different transmit-receive antenna pairs associated with the similar scatterer is less than the tenth of the system sampling period, namely, $\max|\tau_p^{i_1} - \tau_p^{i_2}| \ll T_s$ for $1 \le i_1 \le N_t, 1 \le i_2 \le N_t$, where the system bandwidth is $f_s = 1/T_s$. Consequently, the common sparse pattern is exact in the sense system bandwidth.

III. Pilot Pattern Design

At the receiver, the discrete baseband CIR is

$$\begin{aligned} h[n] &= \tilde{h}(t)\Big|_{t=nT_s} = g_T(t) * h(t) * g_R(t)|_{t=nT_s} \\ &= h(t) * g(t)|_{t=nT_s} = \sum_{p=1}^{P} \alpha_p g(t-\tau_p)|_{t=nT_s}, \end{aligned} \tag{2}$$

where i in (1) is omitted for convenience, $\tilde{h}(t)$ is the analog equivalent baseband CIR by considering the transmit shaping filter $g_T(t)$ and the receive shaping filter $g_R(t)$, $g(t) = g_T(t) * g_R(t)$, and T_s is the sampling period.

The Fourier transform of $h[n]$ is $H(f) = \sum_{k=-\infty}^{\infty} \tilde{H}(f - kf_s)$, and it appears periodic due to the sampling, where $f_s = 1/T_s$ is the system bandwidth. Here the Fourier transform of $\tilde{h}(t)$ is $\tilde{H}(f)$.

We consider $H(f)$ when $k \in [-f_s/2, f_s/2]$, i.e.,

$$H(f) \overset{(a)}{\approx} \tilde{H}(f) = \sum_{p=1}^{P} \alpha_p G(f) e^{-j2\pi f \tau_p} \overset{(b)}{\approx} \sum_{p=1}^{P} \alpha_p C e^{-j2\pi f \tau_p}, \tag{3}$$

where $G(f)$ is the Fourier transform of $g(t)$, the approximation (a) is derived because $G(f)$ has a good suppressed characteristic outside the band, i.e., $G(f) \approx 0$ for $f \notin [-f_s/2, f_s/2]$, and the approximation (b) is valid because $G(f)$ satisfies the constant amplitude within the passband, i.e., $G(f) \approx C$ for $f \in [-f_s/2, f_s/2]$. Here we assume $C = 1$ [8].

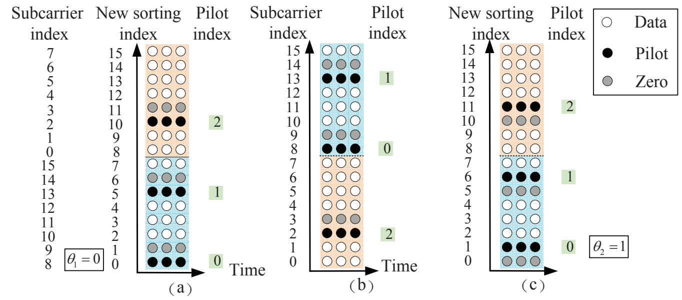

Fig. 2. Pilot pattern design, where $N_t = 2$, $D = 5$, $N = 16$, $N_p = 3$, $N_{p\_\text{total}} = 6$. (a): The first transmit antenna, in the order of the new index; (b): The first transmit antenna, in the order of the subcarrier index; (c): The second transmit antenna, in the order of the new index.

Since $H(f)$ is f_s-periodic, the N-point discrete Fourier transform (DFT) of $h[n]$ can be expressed as

$$\breve{H}[k] = \begin{cases} H(\frac{kf_s}{N}), 0 \le k < \frac{N}{2}, \\ H(-\frac{(N-k)f_s}{N}), \frac{N}{2} \le k < N. \end{cases} \tag{4}$$

From (3), (4), we can observe that the phase of $\breve{H}[k]$ when $k \in [0, N/2-1]$ or $k \in [N/2, N]$ satisfies the linear phase property, respectively, but there is a jump between $k = N/2-1$ and $k = N/2$. We introduce a new index rule $R(k)$ as

$$R(k) = \begin{cases} k + N/2, 0 \le k < N/2, \\ k - N/2, N/2 \le k < N. \end{cases} \tag{5}$$

Under this index sorting rule as shown in Fig. 2 (a) and (b), for the ith transmit antenna, N_p pilots are uniformly spaced with the pilot interval D (e.g., $D = 5$ in Fig. 2). Meanwhile, every pilot is allocated with a pilot index $0 \le l \le N_p - 1$, which is ascending with the increase of the new index. Hence in the sense of the pilot index, the channel frequency responses (CFRs) over pilots satisfy the linear phase property, which is required in the super-resolution estimations of path delays.

Finally, to distinguish channels associated with different transmit antennas, pilots of different transmit antennas use different index initial phase θ_i ($1 \le i \le N_t$) to ensure the orthogonality as shown in Fig. 2 (a) and (c). Hence the total number of pilots is $N_{p\_\text{total}} = N_t N_p$.

IV. Sparse Channel Estimation for Massive MIMO

Druing a certain OFDM symbols, the estimated channel frequency responses (CFRs) of pilots can be written as

$$\hat{\mathcal{H}}^i[l] = \sum_{p=1}^{P} \alpha_p^i e^{-j2\pi \frac{(\theta_i + lD) f_s}{N} \tau_p^i} + W^i[l], 1 \le l \le N_p, \tag{6}$$

where the fast Fourier transform (FFT) size is N, $\hat{\mathcal{H}}^i[l]$ is the estimated CFR of the lth pilot, D is the pilot interval, θ_i is the index initial phase of the ith transmit antenna to ensure the orthogonality of pilot patterns of the different transmit antennas, and $W[l]$ is additive white Gaussian noise (AWGN).

(6) can also be rewritten as

$$\hat{\mathcal{H}}^i[l] = W[l] + \begin{bmatrix} \gamma^{(l-1)D\tau_1^i} & \gamma^{(l-1)D\tau_2^i} & \ldots & \gamma^{(l-1)D\tau_P^i} \end{bmatrix} \begin{bmatrix} \alpha_1^i \gamma^{\theta_i \tau_1^i} \\ \alpha_2^i \gamma^{\theta_i \tau_2^i} \\ \vdots \\ \alpha_P^i \gamma^{\theta_i \tau_P^i} \end{bmatrix}, \tag{7}$$

where $\gamma = e^{-j2\pi \frac{f_s}{N}}$.

Because the wireless channel is inherently sparse and the path delays of different transmit antennas associated with the same receive antenna share the common sparse pattern, i.e., $\tau_p^i = \tau_p$, $1 \le p \le P$, $1 \le i \le N_t$. Hence (7) can be written as

$$\hat{\mathbf{H}} = \mathbf{V}\mathbf{A} + \mathbf{W}, \tag{8}$$

where

$$\hat{\mathbf{H}} = \begin{bmatrix} \hat{\mathcal{H}}^1[0] & \hat{\mathcal{H}}^2[0] & \cdots & \hat{\mathcal{H}}^{N_t}[0] \\ \hat{\mathcal{H}}^1[1] & \hat{\mathcal{H}}^2[1] & \cdots & \hat{\mathcal{H}}^{N_t}[1] \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\mathcal{H}}^1[N_p-1] & \hat{\mathcal{H}}^2[N_p-1] & \cdots & \hat{\mathcal{H}}^{N_t}[N_p-1] \end{bmatrix}, \tag{9}$$

$$\mathbf{V} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \gamma^{D\tau_1^1} & \gamma^{D\tau_2^2} & \cdots & \gamma^{D\tau_P^{N_t}} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma^{(N_p-1)D\tau_1^1} & \gamma^{(N_p-1)D\tau_2^2} & \cdots & \gamma^{(N_p-1)D\tau_P^{N_t}} \end{bmatrix}, \tag{10}$$

$$\mathbf{A} = \begin{bmatrix} \alpha_1^1\gamma^{\theta_1\tau_1^1} & \alpha_1^2\gamma^{\theta_2\tau_1^2} & \cdots & \alpha_1^{N_t}\gamma^{\theta_{N_t}\tau_1^{N_t}} \\ \alpha_2^1\gamma^{\theta_1\tau_2^1} & \alpha_2^2\gamma^{\theta_2\tau_2^2} & \cdots & \alpha_2^{N_t}\gamma^{\theta_{N_t}\tau_2^{N_t}} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_P^1\gamma^{\theta_1\tau_P^1} & \alpha_P^2\gamma^{\theta_2\tau_P^2} & \cdots & \alpha_P^{N_t}\gamma^{\theta_{N_t}\tau_P^{N_t}} \end{bmatrix}, \tag{11}$$

$$\mathbf{W} = \begin{bmatrix} W^1[0] & W^2[0] & \cdots & W^{N_t}[0] \\ W^1[1] & W^2[1] & \cdots & W^{N_t}[1] \\ \vdots & \vdots & \ddots & \vdots \\ W^1[N_p-1] & W^2[N_p-1] & \cdots & W^{N_t}[N_p-1] \end{bmatrix}. \tag{12}$$

It is clear that total least square-ESPRIT (TLS-ESPRIT) algorithm can be directly applied to (8) to estimate the super-resolution path delay if $\mathbf{A}$ is positive definite [13]. However, if it does not satisfy positive definite condition, before ESPRIT method, a spatial smooth technique based additional stage should be performed [13]. In this stage, we obtain the path delay estimation, i.e., $\hat{\tau}_p^i$, $1 \le p \le P$, $1 \le j \le N_r$.

Then the path gains can be acquired by the LS method, i.e.,

$$\hat{\mathbf{A}} = \hat{\mathbf{V}}^{\dagger}\hat{\mathbf{H}} = (\hat{\mathbf{V}}^H\hat{\mathbf{V}})^{-1}\hat{\mathbf{V}}^H\hat{\mathbf{H}}. \tag{13}$$

After acquiring $\hat{\mathbf{A}}$, $\hat{\alpha}_p^i\gamma^{\theta_i\hat{\tau}_p}$ for $1 < i < N_t$ are determined, because θ_i and $\hat{\tau}_p$ are known, we can easily obtain the path gain $\hat{\alpha}_p^{(i,j)}$ for $1 \le p \le P$, $1 \le i \le N_t$, $1 \le j \le N_r$. Consequently, we acquire the estimation of channel parameters, the path delays and path gains.

V. Simulation Results

In this section, we investigated the channel estimation mean square error (MSE) performance in the cyclic prefix (CP)-OFDM for MU massive MIMO-OFDM systems. The conventional comb-type pilot based channel estimation method was selected for comparison. System parameters were set as follows: $f_c = 1800$ MHz, $1/T_s = 20$ MHz, $N = 16384$, $N_g = N/32 = 512$ is the length of the cyclic prefix, which can combat the channel whose $\tau_{\max}$ can be $25.6\ \mu s$. Besides, in the simulation, we adopted a randomized 6 paths channel with the $\tau_{\max} < 25.6\ \mu s$, which is common in classical channel models like International Telecommunication Union Vehicular A and B channel models [14], [15].

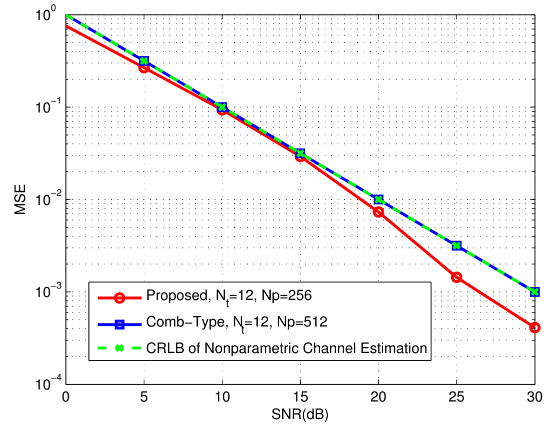

Fig. 3. MSE comparison of the proposed scheme and the conventional scheme.

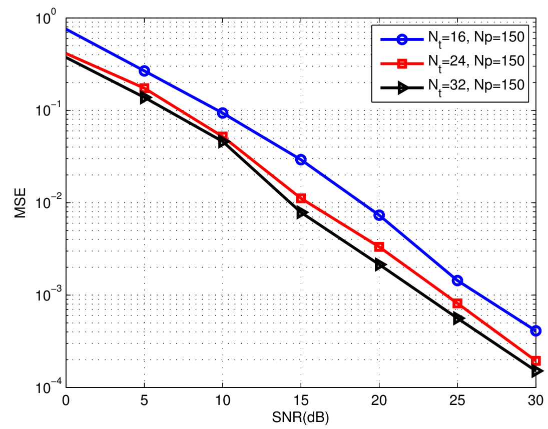

Fig. 4. MSE comparision of the proposed scheme with different number of transmit antennas.

Fig. 3 compared the MSE performance of two schemes, where the Cramer-Rao lower bound (CRLB) of the conventional nonparametric channel estimation scheme [10], [12] is also plotted for comparison. From Fig. 3, we can observe that the proposed scheme is superior to the conventional comb-type scheme. To combat with channels with $\tau_{\max} < 25.6\ \mu s$, the comb-type scheme has to use $N_p = 512$ pilots to ensure the reliable channel recovery. Our proposed scheme exploits the sparsity of the channel. Therefore, the number of pilots in our scheme can be $N_p = 256$, which is less than the conventional scheme obviously. In this way, the proposed scheme enjoys higher spectral efficiency than the conventional schemes. Additionally, it is also observed that, the MSE of the proposed parametric scheme is even superior to the CRLB of the nonparametric channel estimation scheme.

Fig. 4 compared the MSE performance of the proposed schemes with different number of transmit antennas, it can be observed that the MSE performance improves with the increased number of transmit antennas. Equivalently, to obtain the same accuracy of the channel estimation, the pilot overhead can be reduced further. This is because that with the increasing number of transmit antennas, the dimension of measurement matrix $\hat{\mathbf{H}}$ is enlarged, hence the observations used for estimation increase accordingly.

VI. Conclusion

In this paper, a spectrum-efficient parametric channel estimation scheme for MU massive MIMO systems was proposed. In this scheme, we exploit the sparsity and the spatial correlation of the wireless channels to improve the channel estimate as well as reduce the pilot overhead. This channel estimation scheme can achieve the super-resolution estimate of the path delays with arbitrary values. More interestingly, with the increased number of transmit antennas, to acquire the same accuracy channel estimate, the required number of pilots can be reduced further.

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